Quiz 2 Solutions

1. Find all points at which the direction of fastest change of the function $f(x,y) = x^2 + y^2 - 2x - 4y$ is $i + j$.

   Solution. From class we know that a scalar field f undergoes its maximum rate of change in the direction of its gradient vector $\nabla f(x,y)$. Therefore, we have to find all points (x,y) where $\nabla f(x,y)$ is parallel to $i + j$. We can see that

   $$\nabla f(x,y) = (2x-2)i + (2y-4)j.$$

   So we get $(2x-2)i + (2y-4)j = ki + kj$, for some $k > 0$, giving $2x - 2 = 2y - 4$, or $y = x + 1$. In other words, all the points where the direction of fastest change of f is $i + j$ lie on the line $y = x + 1$.

2. Find the points on the hyperboloid $x^2 - y^2 + 2z^2 = 1$ at which the normal line (to the tangent plane) is parallel to the line that joins the points $(3,-1,0)$ and $(5,3,6)$.

   Solution. Let (x', y', z') be any point on the hyperboloid that satisfies the condition given in the problem. Then $\nabla f(x', y', z') = (2x', -2y', 4z')$, and we know that the given line has direction numbers $2, 4, 6$. Therefore, $(2x', -2y', 4z') = k(2,4,6)$, for some $k > 0$, giving $x' = k, y' = -2k$ and $z' = \frac{3}{2}k$. Since (x', y', z') is a point on the hyperboloid, we have that

   $$x'^2 - y'^2 + 2z'^2 = (1 - 4 + \frac{9}{2})k^2 = 1,$$

   so $k = \pm\frac{\sqrt{6}}{3}$. Hence, there there are two possibilities for (x', y', z'), namely $(\pm\frac{\sqrt{6}}{3}, \mp\frac{2\sqrt{6}}{3}, \pm\frac{\sqrt{6}}{2})$.